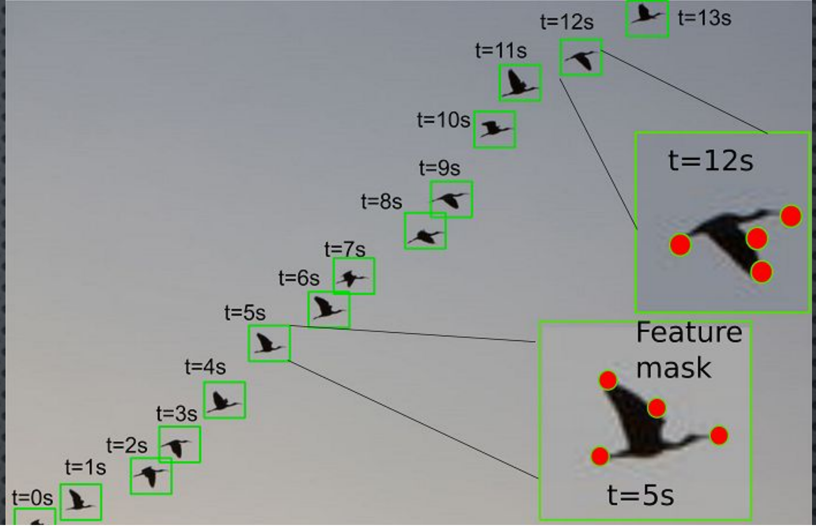
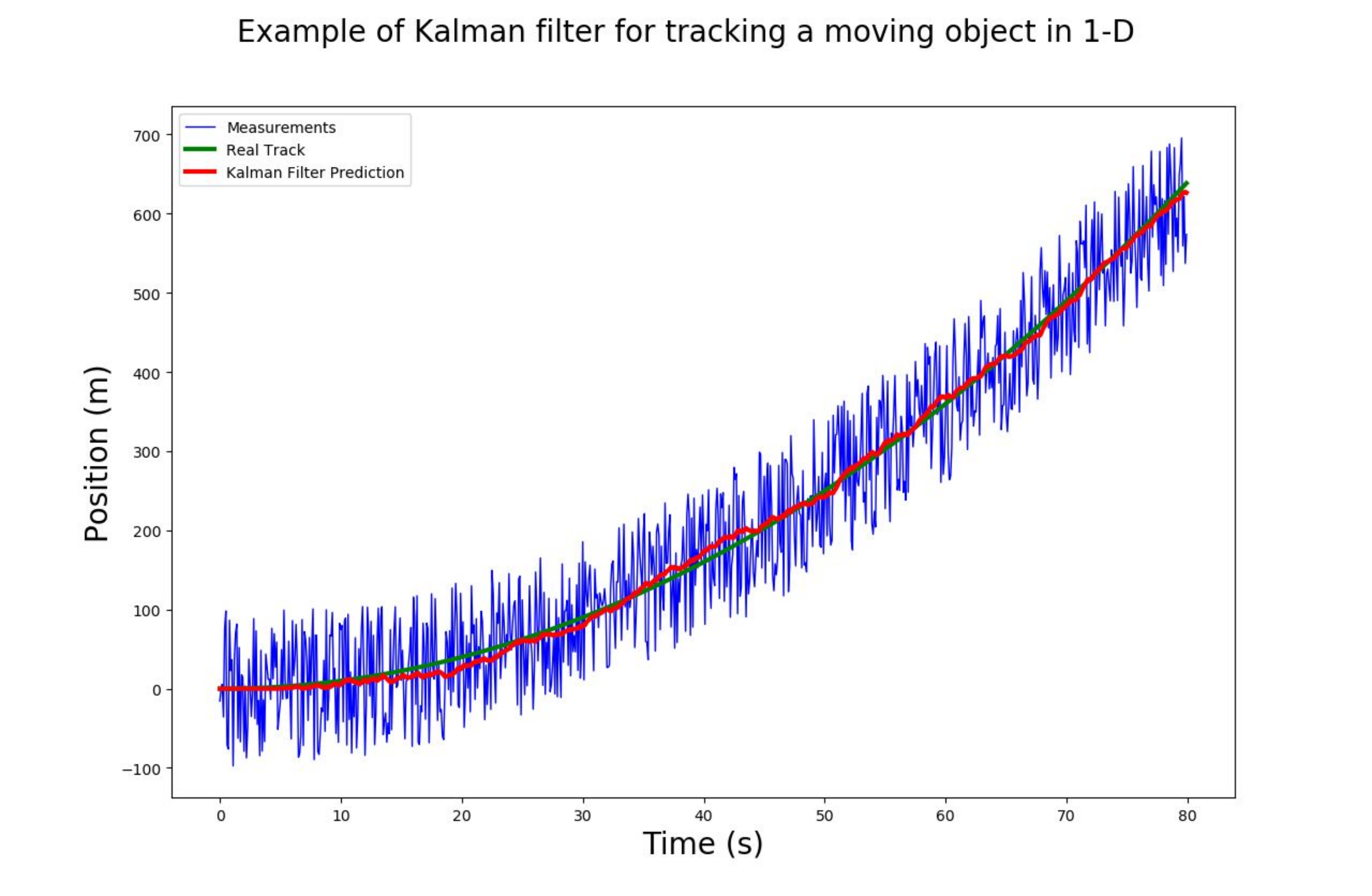
KALMAN FILTERS

WHY?

PREDICTION:

- TEMPERATURE
- ECONOMICS
- ***OBJECT TRACKING***

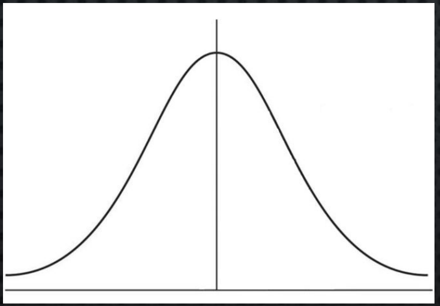




WHAT DO THEY DO?

PROBLEM:

- Given a system with sensors
- The system is linear
- The sensors have Gaussian Noise

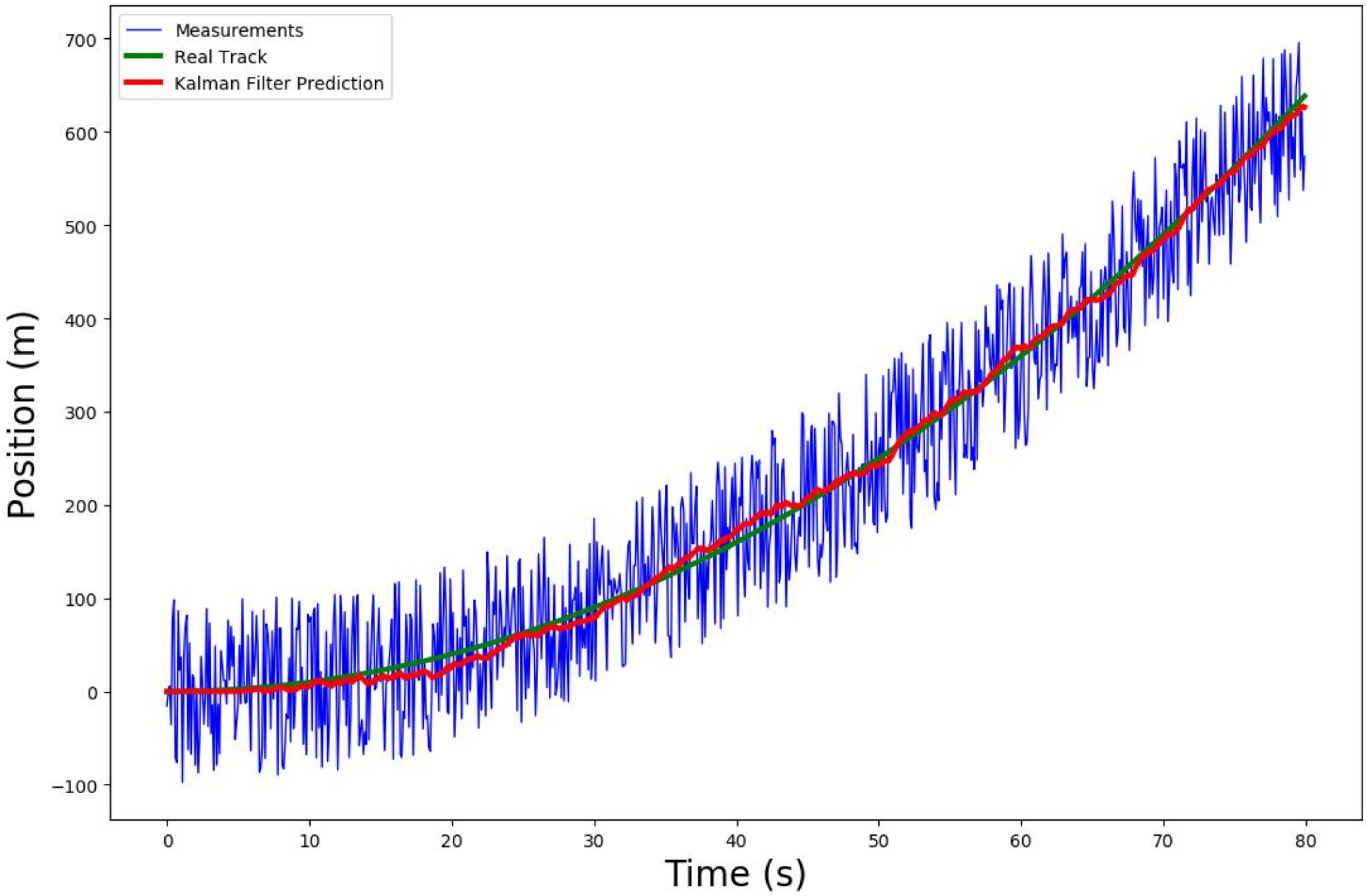


KF SOLUTION:

- Uses previous state of system
  - only needs last state (none before)
- Uses current measurement from sensors
- Combines 2 sources into one output
- This output attempts to eliminate noise from sensors to predict the true state of the system

GOAL: Predict next state of a system

Example of Kalman filter for tracking a moving object in 1-D



WHAT DO THEY DO? PART 2

- Requires 8 parameters:

  1.) F – state transition matrix
  2.) B – control input matrix
  3.) Q – covariance matrix of process noise
  4.) u – control input vector (closely tied with x)
  5.) x_0 – initial state of system
  6.) H – measurement matrix
  7.) R – covariance matrix of measurement noise
  8.) $z_1, \dots, z_k$ – measurements from sensor

Can be split up into 2 parts:

1. Process Model (items 1–4)
2. Measurement Model (items 5–8)

SETTING UP BOTH MODELS

Parameters:

1.) F – state transition matrix

2.) B – control input matrix

3.) Q – covariance matrix of process noise

4.) u – control input vector (closely tied with x)

5.) x_0 – initial state of system

6.) H – measurement matrix

7.) R – covariance matrix of measurement noise

8.) $z_1, \dots, z_k$ – measurements from sensor

Process Model

$$x_{k+1} = Fx_k + Bu_k + w_k$$

- w_k is associated with Q
- It is the process noise vector
- $w_k \sim N\ (0, Q)$

Measurement Model

$$z_{k+1} = Hx_k + v_k$$

- v_k is associated with R
- It is the measurement noise vector
- $v_k \sim N\ (0, R)$

QUICK NOTES BEFORE IMPLEMENTATION

Parameters:

1.) F – state transition matrix
2.) B – control input matrix
3.) Q – covariance matrix of process noise
4.) u – control input vector (closely tied with x)
5.) x_0 – initial state of system
6.) H – measurement matrix
7.) R – covariance matrix of measurement noise
8.) $z_1, \dots, z_k$ – measurements from sensor

- Q and R are the noises
  - They are not known and must be tuned
- The covariances of variables are often 0 in object tracking
  - Spatial dimensions are independent

THE ALGORITHM

STAGE 1 (*Prediction*):

- Predicted State Estimate ($\tilde{x}_k$)

  $$\tilde{x}_k = Fx^{+}_{k-1} + Bu_{k-1}$$

- Predicted Error Covariance (P^{-}_{k})

  $$P^{-}_{k} = FP^{+}_{k-1}F^{+} + Q$$

STAGE 2 (*Update*):

- Measurement Residual ($\tilde{y}_k$)

  $$\tilde{y}_k = z_k - H\tilde{x}_k$$

- Kalman Gain (K_k)

  $$K_k = P^{-}_{k}H^{T}(R + HP^{-}_{k}H^{T})^{-1}$$

- Updated State Estimate (x^{+}_{k})

  $$x^{+}_{k} = \tilde{x}_k + K_k\tilde{y}_k$$

- Updated Error Covariance (P^{+}_{k})

  $$P^{+}_{k} = (I - K_kH)P^{-}_{k}$$

Parameters:

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1D EXAMPLE

GOAL: Predict next x position

State Vector: $\bar{X}_k = \begin{bmatrix} x_k \\ v_k \end{bmatrix}$ ➡ $x_{k+1} = x_k + v_k\Delta t + \frac{1}{2}a\Delta t^2$, $v_{k+1} = v_k + a\Delta t$ ➡ $\bar{X}_{k+1} = \underbrace{\begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}}_{F} \bar{X}_k + \underbrace{\begin{bmatrix} \frac{1}{2}\Delta t^2 \\ \Delta t \end{bmatrix}}_{B} * a$

Measurement Vector: $\bar{Z}_k = [z_k]$ ➡ $\bar{Z}_{k+1} = H\bar{X}_k$ ➡ $\bar{Z}_{k+1} = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{H} \bar{X}_k$

Parameters:

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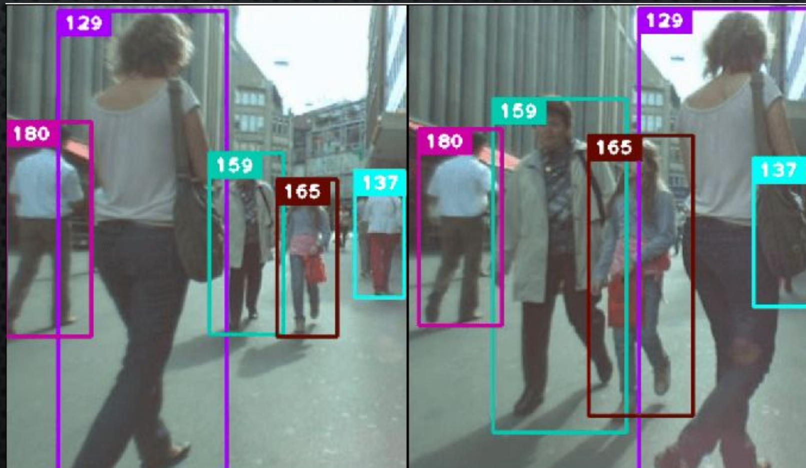
SUMMARY OF K.F. AND ITS CONS

- Uses last state and measurements
- Uses Q and R as the error which are tunable parameters
  - $w_k \sim N\ (0, Q)$
  - $v_k \sim N\ (0, R)$
- Only works if the equation is linear
- Only works if Q and R are Gaussian

* Real-world is non-linear
(i.e. angles of measurements)

Why wouldn't Q and R be Gaussian?

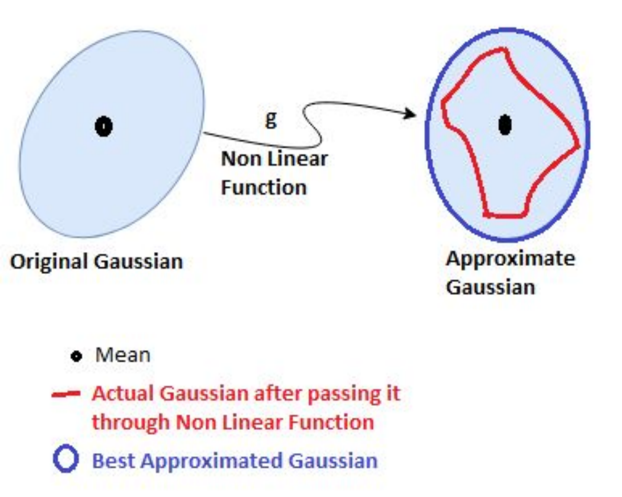
- Obstruction and misdetection of object
- Example:
  - Computer vision detects part of the background as additional part of the object



BEYOND K.F. LIMITATIONS

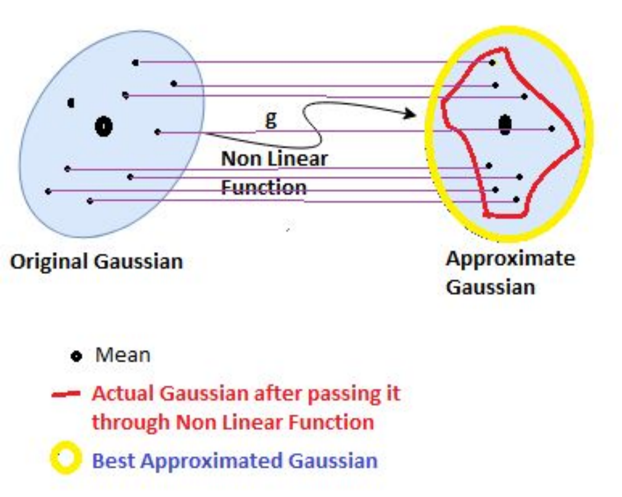
Extended Kalman Filter

- Deals with non-linear problems
- Linearizes the problem
- Does this by approximating around the mean
- Able to use same equations as in Kalman Filter after linearized



Unscented Kalman Filter

- Deals with non-linear problems
- Also linearizes the problem
- Approximates around sigma points
  - One of these points is the mean
  - Each point has an associated weight
- More computationally expensive




"That's all Folks!"